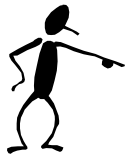
Chapter 1: Introduction

Introduction



The collection processing, interpretation and presentation of numerical data all belong to the domain of statistics. These tasks include the calculation of football goals averages, collecting data on births and deaths, evaluating the effectiveness of commercial products, and forecasting the weather. Statistical information is presented to us constantly on radio and television. Our enthusiasm for statistical facts is encouraged by national newspapers such as the daily journals and magazines.

The word "statistics" is used in several ways. It can refer not only to the mere tabulation of numeric information, as in reports of stock market transactions, but also to the body of techniques used in processing or analyzing data.

The word "statistician" is also used in several ways. The term can be applied to those who simply collect information, as well as to those who prepare analyses or interpretations, and it is also applied to scholars who develop the mathematical theory on which statistics is based.

In Sections 1.1 and 1.2, we discuss the recent growth of statistics and its ever widening range of applications. In Section 1.3 we explain the distinction between the two major branches of statistics, descriptive statistical inference, and in the optional Section 1.4 we discuss the nature of various kinds of data and in connection with this warn the reader against the indiscriminate mathematical treatment of statistical data.

1.1 The Growth of Modern Statistics

The Growth of Modern Statistics

There are several reasons why the scope of statistics and the need to study statistics have grown enormously in the last fifty or so years. One reason is the increasingly quantitative approach employed in all the sciences as well as in business and many other activities which directly affect our lives. This includes the use of mathematical techniques in the evaluation of anti-pollution control, in inventory planning, in the analysis of traffic patterns, in the study of the effects of various kinds of medications, in the evaluation of teaching techniques, in the analysis of competitive behavior of businessmen and governments, in the study of diet and longevity, and so forth. The availability of powerful computers has greatly increased our ability to deal with numerical information. Many types of computers are also inexpensive, so that

sophisticated statistical work can be done by small businessmen, college students, and even high-school students.

The other reason is that the amount of data that is collected, processed, and disseminated to the public for one reason or another has increased almost beyond comprehension, and what part is "good" statistics and what part is "bad" statistics is anybody's guess. To act as watchdogs, more persons with some knowledge of statistics are needed to take an active part in the collection of the data, in the analysis of the data, and what is equally important, in all of the preliminary planning. Without the latter, it is frightening to think of all the things that can go wrong in the compilation of statistical data. The results of costly surveys can be useless if questions are ambiguous or asked in the wrong way, if they are asked of the wrong persons, in the wrong place, or at the wrong time. Much of this is just common sense, as is illustrated by the following examples:

Example 1

Example

To determine public sentiment about the continuation of a certain government program an interviewer asks: "Do you feel that this wasteful program should be stopped?" Explain why this will probably not yield the desired information.

Solution 1

Solution:

The interviewer is "begging the question" by suggesting, in fact, that the program is wasteful.

Example 2

Example

To study consumer reaction to a new convenience food, a house to house survey is conducted during week day mornings, with no provisions for return visits in case no one is home. Explain why this may well yield misleading information.

Solution 2

Solution:

This survey will fail to reach those who are most likely to use the product: single persons and married couples with both spouses employed.

Although much of the above- mentioned growth of statistics began prior to the "computer revolution," the widespread availability and use of computers have greatly accelerated the process. In particular, computers enable us to handle, analyze and dissect large masses of data, and they enable us to perform calculations which previously had been too cumbersome even to contemplate. Our objective in these notes will be your gaining an understanding of the ideas of statistics. Access to a computer is not critical for this objective. Computer uses are occasionally illustrated in these notes, but nearly all the exercises can be done with nothing more than a four – function calculator.

The Study of Statistics

1.2 The Study of Statistics

The subject of statistics can be presented at various levels of mathematical difficulty, and it may be directed toward applications in various fields of inquiry. Accordingly, many textbook have been written on business statistics, educational statistics, medical statistics, psychological statistics … and even on statistics for historians. Although problems arising in these various disciplines will sometimes require special statistical techniques, none of the basic methods discussed in this text is restricted to any particular field of application. In the same way in which 2 + 2 = 4 regardless of whether we are adding dollar amounts, horses, or trees, the methods we shall present provide statistical models which apply regardless of whether the data are IQ's, tax payments, reaction times, humidity readings, test scores, and so on. To illustrate this further, consider the following situations.

1 : In a random sample of 200 retired persons, 137 stated that they prefer living in an apartment to living in a one - family home. At the 0.05 level of significance does this refute the claim that 60 percent of all retired persons prefer living in an apartment to living in a one – family home?

The question asked here should be clear, and it should also be clear that the answer would be of interest mainly to social scientists or to persons in the construction industry. However, if we wanted to cater to the special interests of students of biology, engineering education or ecology, we might rephrase the situation as follows:

2 : In a random sample of 200 citrus trees exposed to a 20^{o} frost, 137 showed some damage to their fruit. At the 0.05 level of significance does this refute the claim that 60 percent of citrus trees exposed to a 20^{o} frost will show some damage to their fruit?

3 : In a random sample of 200 transistors made by a given manufacturer, 137 passed an accelerate performance test. At the 0.05 level of significance does this refute the claim that 60 percent of all transistors made by a given manufacturer will pass the test?

4 : In a random sample of 200 high school seniors in a large city, 137 said that they will go on to college. At the 0.05 level of significance does this refute the claim that 60 percent of all high school seniors in a large city will go to college?

5 : In a random sample of 200 cars tested for the emission of pollutants, 137 failed to meet a state's legal standards. At the 0.05 level of significance does this refute the claim that 60 percent of all cars tested in this state will fail to meet legal emission standards?

So far as the work in these notes is concerned, the statistical treatment of all these versions is the same, and with some imagination the reader should be able to rephrase it for virtually any field of specialization. As some authors do, we could present, and so designate, special problems for readers with special interests, but this would defeat our goal of impressing upon the reader the importance of statistics in all of science, business, and everyday life. To attain this goal we have included in this text exercises covering a wide spectrum of interests.

To avoid the possibility of misleading anyone with our various versions, let us make it clear that we cannot squeeze all statistical problems into the same mold. Although the methods we shall study in these notes are all widely applicable, it is always important to make sure that the statistical model we are using is the right one.

Descriptive Statistics and Statistical Inference

1.3 Descriptive Statistics and Statistical Inference

The origin of modern statistics can be traced to two areas of interest which, on the surface have very little in common: government (political science), and games of chance.

Governments have long used census data to count persons and property, and the problem of describing, summarizing and analyzing census data has led to the development of methods which, until recently, constituted about all there was to the subject of statistics. **These methods, which at first consisted primarily of presenting data in the form of tables and charts, make up what we now call descriptive statistics.** This includes anything done to data which is designed to summarize or describe, without going any further; that is, without attempting to infer anything that goes beyond the data, themselves. For instance, if tests performed on six small cars imported in 1986 showed that they were able to accelerate from 0 to 60 miles per hour in 18.7, 19.2, 16.2, 12.3, 17.5, and 13.9 seconds, and we report that half of them accelerated from 0 to 60 mph in less than 17.0 seconds, our work belongs to the domain of descriptive statistics. This would also be the case if we claim that these six cars averaged

$$\frac{18.7+19.2+16.2+12.3+17.5+13.9}{6}=16.3\,\text{seconds},$$

But is not if we conclude that half of all cars imported that year could accelerate from 0 to 60 mph in less than 17.0 seconds.

Although descriptive statistics is an important branch of statistics and it continues to be widely used, statistics information usually arises from samples (from observations made on only part of a large set of items), and this means that its analysis requires generalizations which go beyond the data. **As a result, the most important feature of the recent growth of statistics has been a shift in emphasis from methods which merely describe to methods which serve to make generalizations; that is, a shift in emphasis from descriptive statistics to the methods of statistical inference.**

Such methods are required, for instance, to predict the operating life span of a hand – held calculator (on the basis of the performance of several such calculators);to estimate the 1995 assessed value of all privately owned property in Cairo (on the basis of business trends, population projections, and so for the); to compare the effectiveness of two reducing diets (on the basis of the weight losses of persons who have been on the diets); to determine the most effective dose of a new medication (on the basis of tests performed with volunteer patients from selected hospitals); or to predict the flow of traffic on a freeway which has not yet been built (on the basis of past traffic counts on alternative routes).

In each of the situations described in the preceding paragraph, there are uncertainties because there is only partial, incomplete, or indirect information; therefore, the methods of statistical inference are needed to judge the merits of our results, to choose a "most promising" prediction, to select a "most reasonable" (perhaps, a "potentially most profitable") course of action.

In view of the uncertainties, we handle problems like these statistical methods which find their origin in games of chance. Although the mathematical study of games of chance dates to the seventeenth century, it was not until the early part of the nineteenth century that the theory developed for "heads or tails," for example, or "red or black" or even or odd," was applied also to real-life situations where the outcomes were "boy or girl," "life or death," "pass or fail," and so forth. Thus, *probability theory was applied to many problems in the behavioral, natural, and social sciences, and nowadays it provides an important tool for the analysis of any situation (in science, in business, or in everyday life) which in some way involves an element of uncertainty of chance. In particular, it provides the basis for the methods which we use when we generalize from observed data, namely, when we use the methods of statistical inference.*

In recent years, it has been suggested that the emphasis has swung too far from descriptive statistics to statistical inference, and that more attention should be paid to the treatment of problems requiring

only descriptive techniques. To accommodate these needs, some new descriptive methods have recently been developed under the general heading of exploratory data analysis. Two of these will be presented.

The Nature of Statistical Data

1.4 The Nature of Statistical Data

Statistical data are the raw material of statistical investigations – they arise whenever measurements are made or observations are recorded. *They may be weights of animals, measurements of personality traits, or earthquake intensities, and they may be simple "yes or no" answer or descriptions of persons' marital status as single, married, widowed, or divorced.* Since we said that statistics deals with numerical data, this requires some explanation, because "yes or no" answers and descriptions of marital status would hardly seem to qualify as being numerical. Observe, however, that we can record "yes or no" answers to a question as 0 (or as 1 and 2, or perhaps as 29 and 30 if we are referring to the 15^{th} "yes or no" question of a test), and that we can record a person's marital status as 1, 2, 3, or 4, depending on whether the person is single, married, widowed, or divorced. In this artificial or nominal way, categorical (qualitative or descriptive) data can be made into numerical data, and if we thus code the various categories, we refer to the numbers we record as nominal data.

Nominal data are numerical in name only, because they do not share any of the properties of the numbers we deal with in ordinary arithmetic. For instance, if we record marital status as 1, 2, 3, or 4, as suggested above, we cannot write 3 > 1 or 2,4, and we cannot write 2 - 1 = 4 - 3, 1 + 3 = 4, or 4 ÷ 2 = 2. It is important, therefore, always to check whether mathematical calculations performed in a statistical analysis are really legitimate.

Let us now consider some examples where data share some, but not necessarily all, of the properties of the numbers we deal with in ordinary arithmetic. For instance, in mineralogy the hardness of solids is sometimes determined by observing "what scratches what." If one mineral can scratch another it receives a higher hardness number, and on Mohs' scale the numbers form 1 to 10 are assigned, respectively, to talc, gypsum, calcite, fluorite apatite, feldspar, quartz, topaz, sapphire, and diamond. With these numbers, we can write 6 > 3, for Example or 7 < 9, since feldspar is harder than calcite and quartz is softer than sapphire. On the other hand, we cannot write 10 - 9 = 2 -1, for Example because the difference in hardness between diamond and sapphire is actually much greater than that between gypsum and talc. Also, it would be meaningless to say that topaz is twice as hard as fluorite simply because their respective hardness numbers on Mohs' scale are 8 and 4.

If we cannot except set up inequalities as was the case in the proceeding example, **we refer to the data as ordinal data.** In connection with ordinal data > does not necessarily mean "greater than" it may be used to denote "happier than," "preferred to," "more difficult than," "tastier than," and so forth.

If we can also form differences, but not multiply or divide, we refer to the data as interval data. To give an example, suppose we are given the following temperature readings Fahrenheit "63^o, 67^o, 91^o, 107^o, 126^o, and 131^o." Here, we can write 107^o is warmer than 68^o and that 91^o is colder than 131^o. Also, we can write $68^o - 63^o = 131^o - 126^o$, since equal temperature differences are equal in the sense that the same amount of heat is required to raise the temperature of an object from 63^o to 68^o as from 126^o to 131^o. On the other hand, it would not mean much if we say that 126^o is twice as hot as 63^o, even though $126 \div 63 = 2$. To show why, we have only to change to the Celsius scale, where the first temperature becomes $\frac{5}{9}(126-32) = 52.2^0$, the second temperature becomes $\frac{5}{9}(63-32) = 17.2^0$, and the first figure is now more than three times the second. This difficulty arises because the Fahrenheit and Celsius scales both have artificial origins (zeros); in other words, the number 0 of neither scale is indicative of the absence of whatever quantity we are typing to measure.

If we can also form quotients, we refer to the data as ratio data, and such data are not difficult to find. They include all the usual measurements (or determinations) of length, height, money amounts, weight, volume, area, pressure, elapsed time (though not calendar time), sound intensity, density, brightness, velocity, and so on.

The distinction we have made here between nominal, ordinal, interval, and ratio data is important, for as we shall see, the nature of a set of data may suggest the use of particular statistical techniques. To emphasize the point that what we can and cannot do arithmetically with a given set of data depends on the nature of the data, consider the following scores which four students obtained in the three parts of a comprehensive history test.

Students	Tests			
	A	B	C	Total
L	89	51	40	180
T	61	56	54	171
H	40	70	55	183
R	13	77	72	162

The totals for the four students are 180, 171, 165, and 162, so that L scored highest, followed by T, H, and R.

Suppose now that somebody proposes that we compare the overall performance of the four students by ranking their scores from high to low for each part of the test, and then average their ranks. What we get is shown in the following table:

	A	**B**	**C**	**Average Rank**
L	1	4	4	3
T	2	3	3	$2\frac{2}{3}$
H	3	2	2	$2\frac{1}{3}$
R	4	1	1	2

Here L's average rank was calculated as $\frac{1+4+4}{3}=\frac{9}{3}=3$, T's as $\frac{2+3+3}{3}=\frac{8}{3}=2\frac{2}{3}$, and so forth.

Now, if we look at the average ranks, we find that R came out best, followed by H, T, and L, so that the order has been reversed from what it was before. How can this be? Well, strange things can happen when we average ranks. For instance, when it comes to their ranks, L's outscoring T by 28 points in counts just as much as T's outscoring her by 5 points in B, and T's outscoring H by 21 points in A counts just as much as H's outscoring him by a single point in C. We conclude that, perhaps, we should not have averaged the ranks, but it might also be pointed out that, perhaps, we should not even have totaled the original scores. The variation of A scores, which go from 13 to 89, is much greater than that of other two kinds of scores, and this strongly affects the total scores and suggests a possible shortcoming of the procedure. We shall not go into this here, as it has been our *goal merely to alert the reader against the indiscriminate use of statistical techniques.*